

Scaling properties of the Lyman- α forest

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Abstract.

We present some statistical features of the large number of Ly α absorption lines detected in high redshift quasar spectra, obtained by using the multifractal approach. In the analysed sample of 12 QSO sight-lines, 11 show scaling behaviour with a crossover between two distinct regimes: a non-homogeneous regime at small scales and a homogeneous regime at large scales. The correlation length shows a redshift dependence, suggesting that the Ly α forest can be an intermediate phenomenon between a strongly inhomogeneous galaxy distribution in the local Universe and a homogeneous initial mass distribution.

1. Introduction

The numerous Ly- α absorption lines seen in quasar spectra can be considered a very deep window on the nature of the young Universe. All recent results obtained with high resolution spectroscopy indicate that the associated gas clouds represent most likely a consistent fraction of the dark side of the baryonic matter physically connected with primeval galaxies. Clustering properties have been studied basically using the two point correlation function. A positive signal was detected in high resolution spectra only for small scales (up to a few hundred km s $^{-1}$, Webb 1987, Rauch et al. 1992, Chernomordik 1995, Cristiani et al. 1996).

The Ly α absorption lines can be fitted by Voigt profiles in order to obtain HI column densities, Doppler widths and redshifts. At high redshift ($z \gtrsim 2$) the redshift evolution and HI column density distribution is well reproduced by a double power law:

$$\frac{\partial^2 n}{\partial z \partial N_{\text{HI}}} = A_o (1+z)^\gamma N_{\text{HI}}^{-\beta} \quad (1)$$

where $\gamma = 2 - 2.6$ and $\beta = 1.4 - 1.7$. The important features of this kind of non Gaussian distribution are the index of the scaling laws and not the amplitudes at every scale and we analyse it using a mathematical tool which is most suitable for their determination.

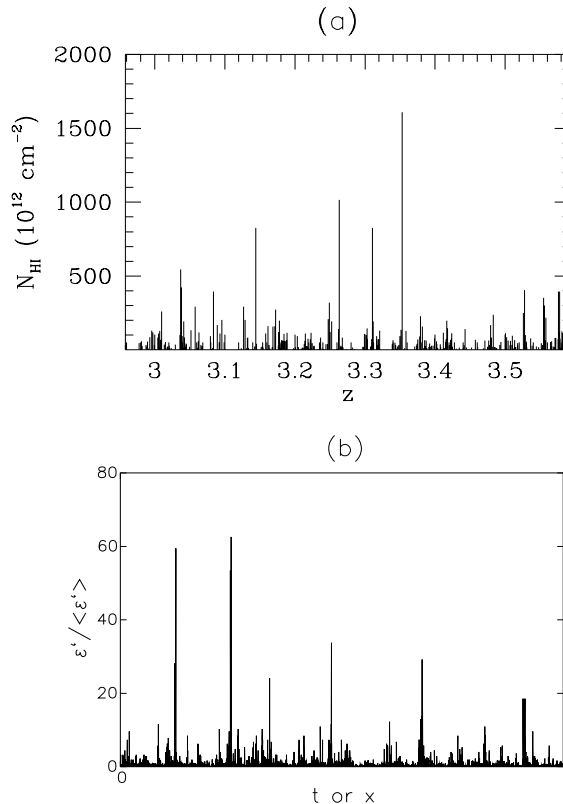


Figure 1. Two examples of multiplicative process. (a) redshift distribution of HI column densities in the spectrum of Q0055 – 26; (b) dissipation rate of the kinetic energy ϵ' in the atmospheric surface layer at a high Reynolds number (see Meneveau & Sreenivasan, 1991).

2. Turbulence and Ly α clouds: The statistic of rare events

In Fig. 1 we show the visual similarity between the distribution of two different physical quantities: the redshift distribution of HI column densities in a quasar spectrum and the kinetic energy dissipation in a fully developed turbulent fluid. The energy transfer, underlying the phenomenon shown in Fig. 1b, can be described by means of a self-similar cascade with an associated multiplicative process: a break-down of large-scale structures into small-scale ones, each receiving a fraction of energy. An analogous mechanism (for instance in a CDM scenario with an associated “inverse cascade” of gravitationally confined structures) can lead to the phenomenon shown in Fig. 1a. The result is an intermittent process, where rare events (the localized peaks, or singularities, in the distribution) have a higher probability to occur with respect to a Gaussian process.

Every scaling process, like turbulence and HI column density distribution in the early Universe, can be treated in the context of the fractal formalism or,

more generally, the multifractal formalism. A multifractal is a scale-invariant distribution described by a local exponent α

$$P_i(r) \sim r^\alpha \quad (2)$$

where P_i is the probability measure in the i -th interval of size r around the point x . In a multifractal, α depends on the position x . To investigate the multifractal structure of the measure, we define the generalized partition function of the box-counting method (Paladin & Vulpiani, 1987):

$$\chi^{(q)}(r) = \sum_i [P_i(r)]^q, \quad (3)$$

where the sum is extended to all the subsets i at a given scale r . The information relative to the multifractal structure can be recognized by calculating the generalized Rényi dimensions D_q from the scaling law:

$$\chi^{(q)}(r) \sim [r]^{(q-1)D_q} \quad (4)$$

For the way the partition function is defined, big values of q emphasize the scaling properties of overdense regions, while small values those of underdense regions. A multifractal structure is a non homogeneous fractal (with different D_q at every q) where the presence of clusters is enhanced by positive values of q and that of voids by negative values. The two point correlation function describes clustering at the first order only, while the infinite set of singularities, each being characterized by a different fractal dimension D_q , describes clustering at every scale and has the important property to show (in principle) the *entire hierarchy of clusters* (if any).

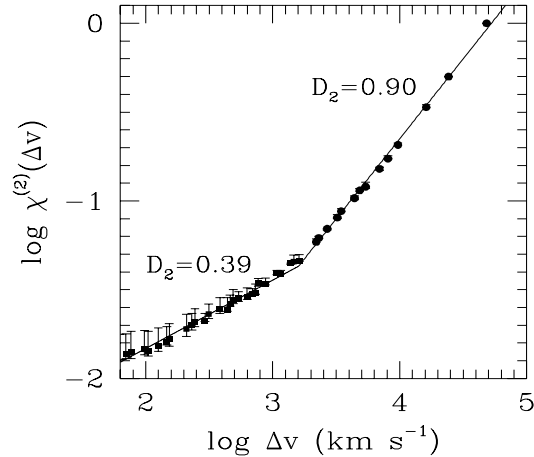


Figure 2. The generalized partition function for $q = 2$ as function of the redshift interval Δv for the HI column densities of Q0055 – 26.

3. Multifractality in Q0055-26

We have used in a previous paper (Carbone & Savaglio, 1996, hereafter CS96) the high resolution spectrum of Q0055-26 ($z_{em} = 3.65$, Cristiani et al. 1995) in order to test the multifractality of the Ly α forest. For each scale Δv (between two redshifts z_1 and z_2) we define a probability measure by dividing the redshift range into disjoint subsets i . This measure $P_i(\Delta v)$ is defined as the total HI column density in the i -th subset characterized by a velocity separation Δv , normalized to the total column density in the spectrum. This can be related to the probability of occurrence of a certain amount of gas in the i -th box at a certain scale Δv .

The results for $q = 2$ are shown in Fig. 2. It is evident the presence of two regimes with a linear relation and two different D_2 values. The separation between the two regimes occurs at $\log \Delta v \simeq 3.2$, which at a mean redshift of $z = 3.305$ corresponds to a comoving scale of about $8 h^{-1}$ Mpc. Similar features are visible for higher values of q up to $q = 4$ (CS96).

For comparison with the distribution of galaxies in the local Universe, one can see Martinez et al. 1990, Coleman & Pietronero 1992, Borgani et al. 1994, Martinez & Coles 1994 and Garrido et al. 1996. In the multifractal analysis of the QDOT redshift survey of 2086 IRAS galaxies (dominated by spiral galaxies), Martinez & Coles (1994) found multifractality and observed two regimes for different values of q . For $q = 2$, at small scales ($r < 10 h^{-1}$ Mpc) they found scaling properties with correlation dimension $D_2^{(3d)} = 2.25$, which in one dimension corresponds to a fractal dimension of 0.25 ($D_2^{(3d)} = D_2^{(1d)} + 2$). For large scales, the IRAS galaxies reach homogeneity. We notice that we are comparing the distribution of galaxies of the local Universe with that of Ly α clouds at high redshift. If the change of regime occurs at similar comoving scales, we conclude that Ly α clouds have undergone a faster clustering evolution compared to IRAS galaxies. However multifractality in galaxies is matter of controversy and it is strongly dependent on the galaxy morphology.

The statistics of the Ly α forest is poor in comparison with galaxy surveys. One of the main problems is thus to test the significance of the results. A first check has been presented in CS96. The observed distribution has been compared with a set of 2000 “fake” distributions. In a following work (Savaglio & Carbone, in preparation), new tests will be presented and sets of different simulations will be compared to the observed distributions. As very preliminary and crude results, we have seen that in 100 simulations of the Q0055-26 sight-line, 53% of the cases shows no scaling law, 37% one single scaling law with $0.58 < D_2 < 0.73$. In 10% a very weak double scaling law, with D_2 which goes from about 0.6 at small scales to about 0.8 at large scales and a correlation length much smaller than the observed one. Even if a genuine multifractality in the Ly α clouds distribution is evident, a richer sample of lines and comparison with simulations would help to clarify this situation.

4. Scaling laws in a sample of QSO spectra

We applied the box-counting method to a sample of QSO spectra. It represents part of the sample used by Cristiani et al. (1996), with a total of 2412 lines

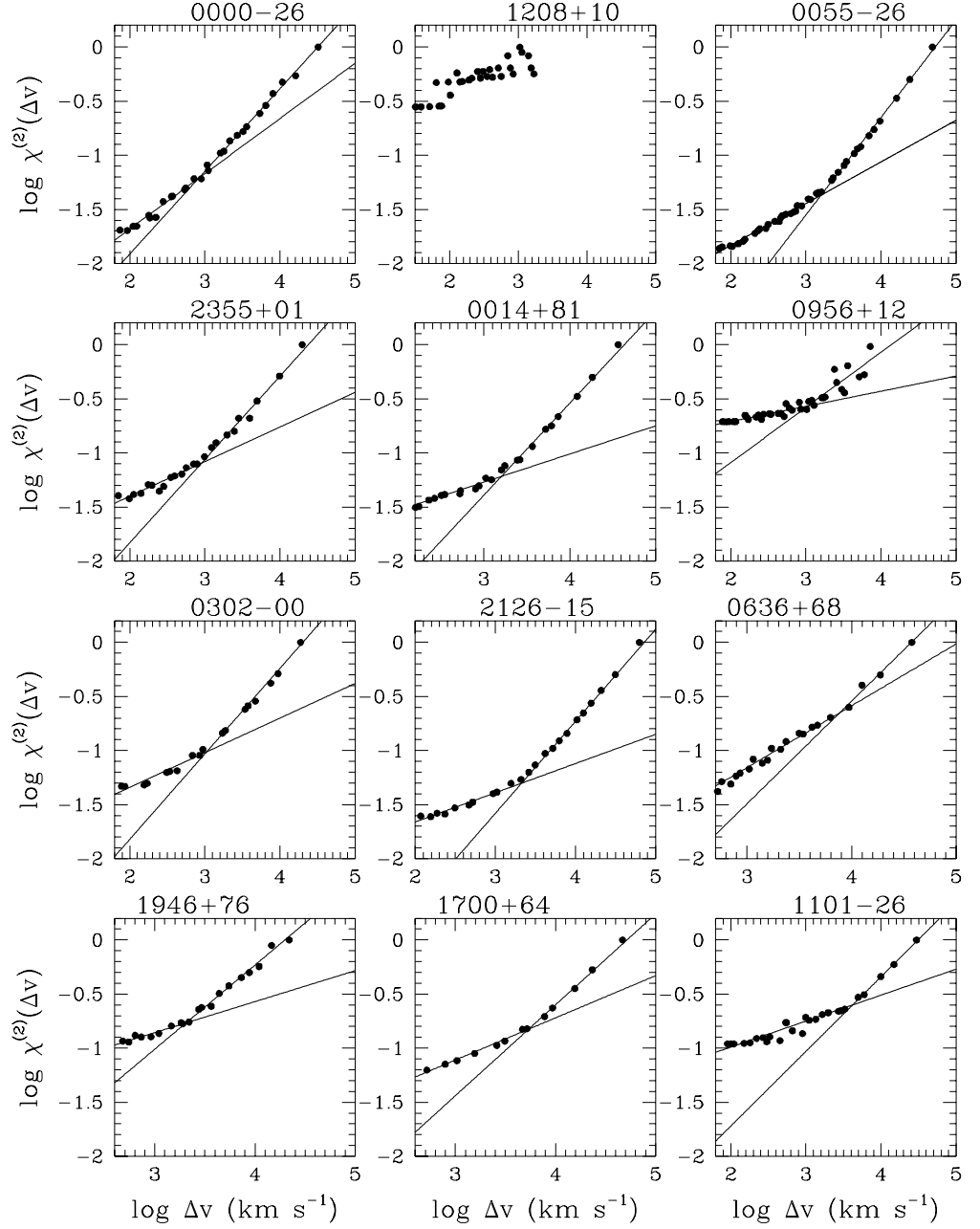


Figure 3. The generalized partition function for $q = 2$ as function of the redshift interval Δv for the HI column densities of 12 QSO sightlines. The plots are in order of decreasing redshift from the top-left to the bottom-right.

for 12 sight-lines. The redshift coverage is $1.85 \lesssim z \lesssim 4.12$ and the resolution better than 15 km s^{-1} , the best available for these sources. The lower limit for the redshift is imposed by the lack of statistics of HST high resolution data. We restricted our analysis to high resolution spectroscopy in order to minimize the problem of line blanketing (the confusion of lines) which at high redshift can dramatically affect the analysis.

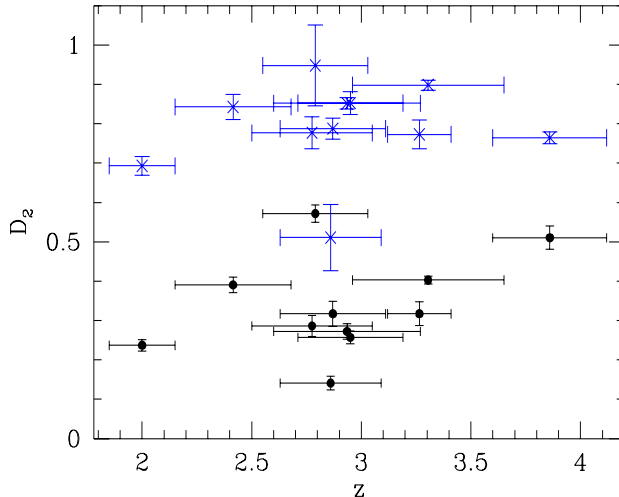


Figure 4. Generalized dimension D_2 as function of redshift for the 11 $\text{Ly}\alpha$ forests in the two regimes of large scales (cross) and small scales (filled circles).

The partition functions with $q = 2$ for the 12 $\text{Ly}\alpha$ forests are shown in Fig. 3. We confirm the presence of two regimes in almost all of them, except for one object ($1208 + 10$), where we do not see any clear scaling law in the distribution. For two objects ($0956 + 12$ and $0636 + 68$) the determination of the two scales is particularly difficult. For the remaining 9 objects, two slopes are clearly visible. In the plot of D_2 as function of the mean redshift for the two different scales (Fig. 4) there is no correlation for large scales, with a mean value of 0.8. This value is an indication that homogeneity has been reached in the sample. For small scales there is a clear deviation from homogeneity. A weak correlation with redshift of D_2 , being smaller for lower redshifts, can also be noticed. A more clear redshift evolution is shown by the distribution of the correlation lengths (Fig. 5). A simple fit with a power law gives:

$$r_o \propto (1 + z)^{-4.5} \quad (5)$$

These results suggest a picture where an initial homogeneous distribution of gas clouds or mass in the Universe is broken by process of fragmentation (in a Cold Dark Matter scenario) or of aggregation of matter around some singularities (in the Hot Dark Matter scenario). The ultimate fate of both the processes is a highly intermittent distribution like that shown in Fig. 1a.

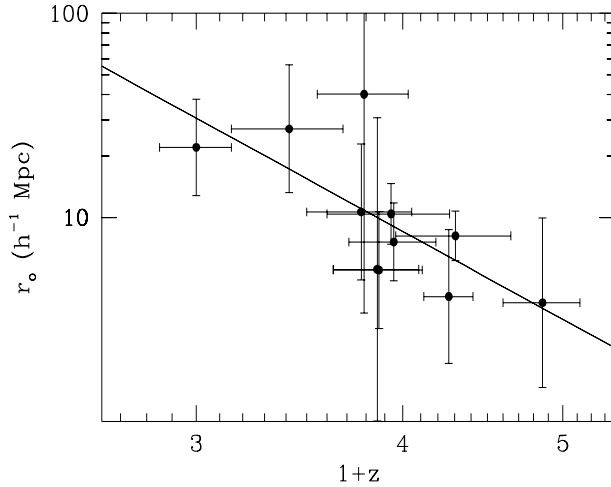


Figure 5. Redshift distribution of the correlation lengths (the scale at which the partition function changes slope). The straight line is the linear regression of the points.

5. Conclusions

The work presented here is in progress. A more extended analysis, with a full description of different methods testing the stability of the results, will be presented elsewhere. Even if the main problem is the lack of statistics, we can firmly conclude that multi-scaling analysis of Ly α forests is a very promising approach to the study of the large scale structure of the Universe at high redshift and its evolution. This has to be regarded as a parallel and complementary point of view with respect to the study of the galaxy distribution.

The two point correlation function analysis can be replaced by different statistics which are more suitable to describe highly inhomogeneous distributions. Ly α clouds have shown scaling laws for much larger scales with respect to previous analysis, around $10 h^{-1}$ Mpc in comoving distance at redshift of about 3 – 3.5. For larger scales, we have no indication against a homogeneous distribution.

The multifractal properties of a sample of 11 quasars show evolution with redshift. In particular both the amplitude and the strength of the multifractality decrease with redshift, which is what one expects to see in a Universe where gravitational clustering gives rise to larger, correlated structures.

These results open the possibility to new scenarios where a local galaxy distribution strongly inhomogeneous up to very large scales, is compatible with a homogeneous initial mass distribution.

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